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Nonstandard Higgs in Electroweak Chiral Lagrangian

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Abstract

We add a nonstandard higgs into the traditional bosonic part of electroweak chiral Lagrangian, in purpose of finding out the contribution to EWCL coefficients from processes with internal line higgs particle. To construct the effective Lagrangian with higgs, we use low energy expansion scheme and write down all the independent terms conserving $SU(2)\times U_Y(1)$ symmetry in the nonlinear representation which we show is equivalent to the linear representation. Then we integrate out higgs using loop expansion technique at 1-loop level, contributions from all possible terms are obtained. We find up to order of p^4 in low energy expansion, three terms, $\mathcal{L}_5$, $\mathcal{L}_7$, $\mathcal{L}_{10}$ in EWCL are important, for which the contributions from higgs can be further expressed in terms of higgs partial decay width $\Gamma_{h\to ZZ}$ and $\Gamma_{h\to WW}$. Higg mass dependence of the coefficients in EWCL are discussed.

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I. INTRODUCTION

The bosonic part of electroweak chiral Lagrangian (EWCL) first introduced by A.Longhitano, T.Appelquist and Bernard in Ref. [1, 2] is used to describe the impact of higgs sector on the rest of the gauge theory, which supposes higgs heavy and disappear from the electroweak interaction at low energy scale $E \ll 1Tev$. The most general form[3] of this EWCL up to order of p^4 includes sixteen independent terms which conserves $SU(2) \times U_Y(1)$ symmetry and contribute to electroweak gauge boson self energy and vertices[3, 4]. Because there are only goldstones and gauge bosons in this effective theory, it provides an economical phenomenological description of electroweak physics below Tev energy scale. The importantce of this higgsless description of EW interaction is based on the fact that higgs has not been found in present experiments below LEP bound $m_h > 114.4Gev$[5] for which the situation will last until the higgs or other new particles are found in future and the fact that the description of higgs sector in SM is actually problematic[6, 7]. Nowadays, in the situation that LHC is going to run in 2007 and ILC is under active discussion, EWCL plays more special role in particle physics, since even with most optimistic estimation, discovery of new particles on LHC needs at least three more years from now on. Before that time, the only correct theory verified by experiments is this higgsless description of EW interaction. In next few years, if we are not lucky in finding new particles on LHC, EWCL may last even more time. During the time that EWCL keep to be correct, the important question we need to answer is: Can we test effects of new particles below their thresholds? In the language of EWCL is to investigate the role of new particles on the coefficients in the EWCL. Since the most interesting new particle is higgs, in this paper, we focus our attentions on investigating effects of single higgs in EWCL, the effects from other possible new particles will be discussed elsewhere. At present stage, due to lack of experiment data, it is impossible to give these effects the quantitative estimations, but we hope through our work, some qualitative features can be evaluated out.

The possibility that an elementary higgs particle with mass higher than 114.4 Gev always stands. Although we still know little about the origin of the spontaneously electroweak symmetry breaking, higgs mechanism tells that a heavy particle with nonzero vacuum expectation value can always help us out of that confusion. Hence it is interesting to consider the possibility that a nonstandard higgs exists at $E > 114.4$ Gev and to find out its possible interactions with the rest of gauge theory. Once a effective theory with higgs included is established above some energy scale, we can in principle integrate out all contributions from heavy higgs to obtain an effective Lagrangian below that scale, i.e., the EWCL. The effects from contributions of the integrated out heavy higgs resides in the coefficients of the EWCL, which reflect effective interactions among goldstones and gauge bosons.

In literature, there are papers to discuss the issue we are interested [8, 9, 10, 11]. The theory for higgs they started with are limited either in standard model (SM) or some simplified higgs models which has no custodial symmetry breaking. In this paper, we start from the most general theory–EEWCL which is an extended EWCL theory with higgs included in to make our investigations. This article is organized as follows: in Sec.II, two different descriptions of the EW interaction, linear and nonlinear representations, are discussed and the equivalence between these two is demonstrated. In Sec.III, all possible independent interaction terms among higgs, goldstones and gauge bosons are introduced and a complete set EEWCL up to certain order of the low energy expansion which will finally contribute to p^4 order EWCL at one loop level is constructed. After that, in Sec.IV, we use loop expansion scheme[12] to integrate out the heavy higgs to obtain the contributions from it to the traditional EWCL. Finally, some integration results are listed and discussed.

II. EQUIVALENCE BETWEEN LINEAR AND NON-LINEAR REPRESENTATION

Higgs is first introduced into electroweak sector by Weinberg and Salam[13] as a SU(2) isospin doublet,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \tag{1}$$

which transforms linearly under $SU(2)$ and $U_Y(1)$ gauge group action. And all the interactions related to higgs doublet can be constructed readily with simple lie algebras. With this linear form of higgs doublet, the bosonic part of higgs effective Lagrangian can be written as

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_n \frac{f_n}{\Lambda_H^2} \mathcal{O}_n \ . \tag{2}$$

The SM Lagrangian contains terms up to dimension four operators, thus nonstandard higgs interaction starts from dimension six. In dimension six part of Lagrangian, 12 independent operators form a basis set. In the notation of Ref. [14, 15, 16], they are

$$
\begin{aligned}
\mathcal{O}_{DW} &= Tr([D_\mu, \hat{W}_{\nu\rho}][D^\mu, \hat{W}^{\nu\rho}]) \\
\mathcal{O}_{DB} &= -\frac{g'^2}{2}\partial_\mu B_{\nu\rho}\partial^\mu B^{\nu\rho} \\
\mathcal{O}_{BW} &= \Phi^+ \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi \\
\mathcal{O}_{\Phi,1} &= \left[(D_\mu\Phi)^+\Phi\right]\left[\Phi^+ D^\mu\Phi\right] \\
\mathcal{O}_{WWW} &= Tr(\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}_\rho^\mu) \\
\mathcal{O}_{WW} &= \Phi^+\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi \\
\mathcal{O}_{BB} &= \Phi^+\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi \\
\mathcal{O}_{W} &= (D_\mu\Phi)^+\hat{W}^{\mu\nu}(D_\nu\Phi) \\
\mathcal{O}_{B} &= (D_\mu\Phi)^+\hat{B}^{\mu\nu}(D_\nu\Phi) \\
\mathcal{O}_{\Phi,2} &= \frac{1}{2}\partial_\mu(\Phi^+\Phi)\partial^\mu(\Phi^+\Phi) \\
\mathcal{O}_{\Phi,3} &= \frac{1}{3}(\Phi^+\Phi)^3 \\
\mathcal{O}_{\Phi,4} &= (\Phi^+\Phi)\left[(D_\mu\Phi)^+(D^\mu\Phi)\right] \ .
\end{aligned} \tag{3}
$$

The covariant derivative D is given by

$$
D_\mu = \partial_\mu + igT^aW_\mu^a + ig'YB_\mu \ , \tag{4}
$$

where g is SU(2) coupling with $Tr(T^aT^b) = \frac{1}{2}\delta^{ab}$, g' is $U_Y(1)$ coupling and Y is the hypercharge operator. Define

$$
\hat{W}_{\mu\nu} = igT^aW_{\mu\nu}^a \qquad\qquad \hat{B}_{\mu\nu} = ig'B_{\mu\nu} \ , \tag{5}
$$

hence

$$
[D_\mu, D_\nu] = \hat{W}_{\mu\nu} + \hat{B}_{\mu\nu} \ . \tag{6}
$$

Using these twelve operators, effective interactions between higgs doublet field and gauge bosons could be calculated directly. Operators in dimension 8 are given in Ref. [17].

Different from the doublet representation, EW chiral Lagrangian(EWCL)[1, 2, 3] takes a non-linear representation, with goldstone field given by,

$$
U = e^{i\frac{\tau^i\pi^i}{2f}}, \qquad\qquad i = 1, 2, 3 \tag{7}
$$

where $\tau^i, i = 1, 2, 3$ are three pauli matrices. In the notation of Ref.[2], there are two independent terms in p^2 order,

$$\mathcal{L}^{(2)} = -\frac{f^2}{4} Tr[(D_\mu U)^+ (D^\mu U)] + \frac{\beta_1 f^2}{4} [Tr(TV_\mu)]^2 \ , \tag{8}$$

where

$$D_\mu U = \partial_\mu U + ig\frac{\tau^a}{2} W^a_\mu U - ig' U \frac{\tau^3}{2} B_\mu \ . \tag{9}$$

T operator in the second terms of (8) breaks custodial symmetry into $SU(2) \times U_Y(1)$ even in the absence of $U_Y(1)$ gauge coupling, with

$$\begin{aligned} & V_\mu = (D_\mu U) U^+ \qquad T = U\tau^3 U^+ \\ & W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu] \\ & B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \ . \end{aligned} \tag{10}$$

All these four building operators in (10) are $SU(2)$ covariant and $U_Y(1)$ invariant. On this basis, fourteen independent terms in p^4 order are given by,

$$\begin{aligned}
\mathcal{L}_1 &\equiv \frac{\alpha_1}{2} g g' B_{\mu\nu} Tr(TW^{\mu\nu}) = \frac{\alpha_1}{2} g g' l_4^1 \\
\mathcal{L}_2 &\equiv \frac{i\alpha_2}{2} g' B_{\mu\nu} Tr(T[V^\mu, V^\nu]) = \frac{i\alpha_2}{2} g' l_4^2 \\
\mathcal{L}_3 &\equiv i\alpha_3 g Tr(W_{\mu\nu}[V^\mu, V^\nu]) = i\alpha_3 g l_4^3 \\
\mathcal{L}_4 &\equiv \alpha_4 [Tr(V_\mu V_\nu)]^2 = \alpha_4 l_4^4 \\
\mathcal{L}_5 &\equiv \alpha_5 [Tr(V_\mu V^\mu)]^2 = \alpha_5 l_4^5 \\
\mathcal{L}_6 &\equiv \alpha_6 Tr(V_\mu V_\nu) Tr(TV^\mu) Tr(TV^\nu) = \alpha_6 l_4^6 \\
\mathcal{L}_7 &\equiv \alpha_7 Tr(V_\mu V^\mu) Tr(TV_\nu) Tr(TV^\nu) = \alpha_7 l_4^7 \\
\mathcal{L}_8 &\equiv \frac{1}{4}\alpha_8 g^2 [Tr(TW_{\mu\nu})]^2 = \frac{1}{4}\alpha_8 g^2 l_4^8 \\
\mathcal{L}_9 &\equiv \frac{i}{2}\alpha_9 g Tr(TW_{\mu\nu}) Tr(T[V^\mu, V^\nu]) = \frac{i}{2}\alpha_9 g l_4^9 \\
\mathcal{L}_{10} &\equiv \frac{1}{2}\alpha_{10} [Tr(TV_\mu) Tr(TV_\nu)]^2 = \frac{1}{2}\alpha_{10} l_4^{10} \\
\mathcal{L}_{11} &\equiv \frac{1}{2}\alpha_{11} g \epsilon_{\mu\nu\rho\lambda} Tr(TV^\mu) Tr(V^\nu W_{\rho\lambda}) = \frac{1}{2}\alpha_{11} g l_4^{11} \\
\mathcal{L}_{12} &\equiv \alpha_{12} g Tr(TV^\mu) Tr(V_\nu W^{\mu\nu}) = \alpha_{12} g l_4^{12} \\
\mathcal{L}_{13} &\equiv \alpha_{13} g g' \epsilon_{\mu\nu\rho\lambda} B^{\mu\nu} Tr(TW^{\rho\lambda}) = \alpha_{13} g g' l_4^{13} \\
\mathcal{L}_{14} &\equiv \alpha_{14} g^2 \epsilon_{\mu\nu\rho\lambda} Tr(TW^{\mu\nu}) Tr(TW^{\rho\lambda}) = \alpha_{14} g^2 l_4^{14} \ .
\end{aligned} \tag{11}$$

$\mathcal{L}_{12}$, $\mathcal{L}_{13}$ and $\mathcal{L}_{14}$ are three CP violation terms.

Actually, there is a natural connection between the doublet linear representation given in (3) and Non-linear representation given in (11), which we would show explicitly below. In unitary gauge, higgs doublet is parameterized as

$$\Phi = \frac{1}{\sqrt{2}} e^{i\pi^a\tau^a} \begin{pmatrix} 0 \\ h+v \end{pmatrix} . \tag{12}$$

Define the charge conjugation of Φ

$$\Phi^c \equiv i\tau^2\Phi^* = \begin{pmatrix} h+v \\ 0 \end{pmatrix} . \tag{13}$$

Set

$$\Sigma \equiv \begin{pmatrix} \Phi^c & \Phi \end{pmatrix} = \frac{h+v}{\sqrt{2}} e^{i\pi^a\tau^a} \equiv \frac{h+v}{\sqrt{2}} U , \tag{14}$$

where U is defined as

$$U \equiv e^{i\pi^a\tau^a} . \tag{15}$$

By doing some SU(2) algebras, we find following connections between the two representations

$$\begin{aligned}
2(D_\mu\Phi)^+\Phi &= \partial_\mu h^2 + h^2 Tr(TV_\mu) \\
2\Phi^+ W_{\mu\nu}\Phi &= h^2 Tr(TW_{\mu\nu}) \\
2(D_\mu\Phi)^+(D_\nu\Phi) &= h^2[Tr(TV_\mu V_\nu) - Tr(V_\mu V_\nu)] + 2(\partial_\mu h)(\partial_\nu h) \\
2(D_\mu\Phi)^+ W^{\mu\nu}(D_\nu\Phi) &= h^2 Tr(W^{\mu\nu}V_\mu V_\nu) - (\partial_\mu h^2) Tr(W^{\mu\nu}V_\nu) \\
2\Phi^+ W^{\nu\rho}(D^\mu\Phi) &= h^2[Tr(TV^\mu W^{\nu\rho}) + Tr(V^\mu W^{\nu\rho})] \\
2(D^\mu\Phi)^+ W^{\nu\rho}\Phi &= h^2[Tr(TV^\mu W^{\nu\rho}) - Tr(V^\mu W^{\nu\rho})] .
\end{aligned} \tag{16}$$

Here higgs field h and goldstone field U are defined as

$$h^2 \equiv \det\Sigma \qquad \Sigma \equiv hU . \tag{17}$$

Thus higgs here is a $SU(2)\times U(1)$ scalar, denoting the module freedom of higgs doublet, while goldstone U denotes the rotation angle of EW gauge transformation, which is similar to the case in the chiral Lagrangian of strong interaction, where a scalar meson σ denotes the module and eight goldstone U denotes angle of the strong chiral transformation[18]. Note that h is not exactly the parameter $'h'$ of higgs doublet in the unitary gauge, but differs in a factor of $\sqrt{2}$.

We can also express all possible interacting terms in the non-linear representation with higgs doublet as followings:

$$\begin{aligned}
&Tr(TV_\mu) = (\Phi^+\Phi)^{-1}[2(D_\mu\Phi)^+\Phi - \partial_\mu(\Phi^+\Phi)]\\
&Tr(TW_{\mu\nu}) = 2(\Phi^+\Phi)^{-1}[\Phi^+W_{\mu\nu}\Phi]\\
&Tr(V_\mu V_\nu) = \frac{1}{2}(\Phi^+\Phi)^{-2}\partial_\mu(\Phi^+\Phi)\partial_\nu(\Phi^+\Phi) - (\Phi^+\Phi)^{-1}[(D_\mu\Phi)^+(D_\nu\Phi) + h.c.]\\
&Tr(TV_\mu V_\nu) = (\Phi^+\Phi)^{-1}[(D_\mu\Phi)^+(D_\nu\Phi) - h.c.]\\
&Tr(V^\mu W^{\nu\rho}) = (\Phi^+\Phi)^{-1}[-(D^\mu\Phi)^+W^{\nu\rho}\Phi + h.c.]\\
&Tr(TV^\mu W^{\nu\rho}) = (\Phi^+\Phi)^{-1}[(D^\mu\Phi)^+W^{\nu\rho}\Phi + h.c.]\\
&Tr(W^{\mu\nu}V_\mu V_\nu) = 2(\Phi^+\Phi)^{-1}[(D_\mu\Phi)^+W^{\mu\nu}(D_\nu\Phi)] + (\Phi^+\Phi)^{-2}\partial_\mu(\Phi^+\Phi)[-(D^\mu\Phi)^+W^{\nu\rho}\Phi + h.c.]
\end{aligned} \tag{18}$$

In the path integral system, the change of variables induces a determinant factor to the generating functional $\mathcal{Z}$,

$$\mathcal{Z} \;=\; \int \mathcal{D}W_\mu\mathcal{D}B_\mu\mathcal{D}U\mathcal{D}h\,\exp\{iS'[W_\mu^a, U, h]\}\,\det\{i\delta^{(4)}(0)(h+v)\} \tag{19}$$

The determinant can be written in the exponential form. Correspondingly, the lagrangian density transforms as

$$\mathcal{L} \to \mathcal{L}' + \delta^{(4)}(0)\ln{(h+v)} \tag{20}$$

This determinant which contains quartic divergences is necessary to cancel exactly the quartic divergences brought into by the longitudinal part of gauge boson.[19, 20] However in our discussion the logarithm term is totally absorbed in the free parameters of the higgs potential.

According to above facts and algebra relations, any terms written in a linear representation with doublet Φ can be transformed into terms written in a nonlinear one with scalar h and U, and vice verse. In this sense, an effective theory written in two different representations are actually the same. For example, bosonic sector of SM could be transformed as

$$\mathcal{L}_{SM} = \frac{1}{2}(\partial_\mu h)^2 - \frac{(h+v)^2}{4}Tr(V_\mu V^\mu) + \mu^2(h+v)^2 - \lambda(h+v)^4\,. \tag{21}$$

III. AN EXTENDED EW CHIRAL LAGRANGIAN (EEWCL)

From relations in (16), we know higgs could be added into traditional EW chiral Lagrangian in a simple way. According to the principle of phenomenological Lagrangian by Weinberg in Ref. [21], We just write down all possible independent terms conserving $SU(2) \times U(1)$ symmetry, and arrange them order by order. The construction of the extended EW chiral Lagrangian can be done in quite a straight way, once we know how to deal with higgs in power counting. Traditional EWCL takes Weinberg's power counting scheme[21], in which goldstone U is counted as p^0 order, while ∂_μ and gauge boson fields as p^1 order. But here we have no good reason to designate higgs any certain power, in that we suppose higgs interacts with gauge bosons in arbitrary way permitted by $SU(2) \times U(1)$ symmetry. Because our purpose is to find out the effective contribution from higgs to EWCL couplings, thus a better way is to take a expansive view on this heavy higgs field, i.e.

$$h = h^{(0)} + h^{(2)} + h^{(4)} + \dots \tag{22}$$

However, at present we needn't care much about this power counting problem, as long as all terms which would contribute to EWCL couplings are included. Through complicated calculation, following terms are found out to be complete and independent,

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \mathcal{L}^{(4)} \\
\mathcal{L}^{(at\ least\ 0)} &= -V(h) \\
\mathcal{L}^{(at\ least\ 2)} &= \frac{1}{2}(\partial_\mu h)^2 + C_1(h)A_\mu^2 + C_2(h)tr(V_\mu^2) \\
\mathcal{L}^{(at\ least\ 4)} &= C_3^i(h)l_4^i + C_4^j(h)(\partial_\mu h)l_3^{j\mu} + C_5^k(h)(\partial_\mu h)(\partial_\nu h)l_2^{k\mu\nu} \\
&\quad +C_6(h)(\partial_\mu h)^2(\partial_\nu h)A^\nu + C_7(h)(\partial_\mu h)^4 \\
\mathcal{L}^{(at\ least\ 6)} &= C_8^l(h)(\partial_\mu h)(\partial_\nu h)l_4^{l\mu\nu} \ ,
\end{aligned} \tag{23}$$

in which

$$A_\mu = tr(TV_\mu) \ , \tag{24}$$

$V(h)$ is some arbitrary potential of h, $C_n(h)$ are coefficient functions depending on higgs field. $l_4^i, i = 1, 2, \dots, 14$, are p^4 order operators defined in (11), $l_3^{j\mu}, j = 1, 2, \dots, 7$ and $l_2^{k\mu\nu}, k = 1, 2$, are p^3 and p^2 order tensors depending on goldstone U, gauge field W and B, they are given by

$$
\begin{array}{lll}
l_2^{1\mu\nu} = Tr(TV^\mu)Tr(TV^\nu) & l_2^{2\mu\nu} = Tr(V^\mu V^\nu) & \\
l_3^{1\mu} = Tr(TV^\mu)Tr(V^\nu V_\nu) & l_3^{2\mu} = Tr(TV^\nu)Tr(V^\mu V_\nu) & \\
l_3^{3\mu} = Tr(TV^\nu)Tr(TV^\mu V_\nu) & l_3^{4\mu} = Tr(TV_\nu)Tr(TW^{\mu\nu}) & \\
l_3^{5\mu} = B^{\mu\nu}Tr(TV_\nu) & l_3^{6\mu} = Tr(TW^{\mu\nu}V_\nu) & l_3^{7\mu} = Tr(W^{\mu\nu}V_\nu)\ .
\end{array}
$$

The last term $l_4^{l\mu\nu}$ includes all possible forms of tensors with symmetric $\mu\nu$ indices in p^4 order.

$$
\begin{aligned}
l_4^{1\mu\nu} &= B^\mu_\rho Tr(TW^{\nu\rho}) \\
l_4^{2\mu\nu} &= B^\mu_\rho Tr(T[V^\nu, V^\rho]) \\
l_4^{3\mu\nu} &= Tr(W^\mu_\rho [V^\nu, V^\rho]) \\
l_4^{4\mu\nu} &= Tr(V^\mu V_\rho)Tr(V^\nu V^\rho) \\
l_4^{5\mu\nu} &= Tr(V^\mu V^\nu)Tr(V^\rho V_\rho) \\
l_4^{6\mu\nu} &= Tr(V^\mu V^\nu)Tr(TV^\rho)Tr(TV_\rho) \\
l_4^{7\mu\nu} &= Tr(V^\mu V_\rho)Tr(TV^\nu)Tr(TV^\rho) \\
l_4^{8\mu\nu} &= Tr(TW^\mu_\rho)Tr(TW^{\mu\rho}) \\
l_4^{9\mu\nu} &= Tr(TW^\mu_\rho)Tr(T[V^\mu, V^\rho]) \\
l_4^{10\mu\nu} &= Tr(TV^\mu)Tr(TV^\nu)Tr(TV^\rho)Tr(TV_\rho) \\
l_4^{12\mu\nu} &= Tr(TV^\mu)Tr(V_\rho W^{\nu\rho})\ .
\end{aligned} \tag{25}
$$

Two terms $C_9(h)Tr(W_{\mu\nu}W^{\mu\nu})$ and $C_{10}(h)B_{\mu\nu}B^{\mu\nu}$ are not included in (23) since their contributions could be represented by redefinition of gauge boson field and the gauge coupling[15].

In (23), besides the old terms l_4^{11},l_4^{12},l_4^{13}, there are other new terms violating CP, whose couplings are C_6, C_4^3, C_4^4, C_4^6 and C_8^{12}, respectively. It is interesting that these CP violating terms might contribute to CP conserving effective couplings in EWCL.

IV. INTEGRATE OUT HIGGS

Suppose higgs is heavy, we want to integrate out higgs in this EEWCL to obtain the contributions to the effective couplings of the rest part of EW interaction. Because we stick to the equivalence between the linear and non-linear representation, we need a nonzero vacuum condensation of higgs field to induce mass to gauge bosons. When all external fields V_μ, W_μ and B_μ vanish, we are left with a theory of higgs self-interaction. The vacuum expectation value(vev) is determined by the stationery equation of higgs potential

$$0 = \delta V(h) \Rightarrow V'(h)\Big|_{h=v} = 0 \ . \tag{26}$$

If we first complete the renormalization of this higgs potential, then find the solution of the stationery equation of the renormalized potential, we get the physical vev of higgs which includes contributions from all quantum corrections. We take v here a free parameter because higgs potential itself is totally free. And the physical higgs $\tilde{h}$ with zero vev is given by

$$h = \tilde{h} + (v + \delta v) \ , \tag{27}$$

here δv denotes loop correction to vev, which is determined by

$$\frac{d}{dh}[V_{tree}(h) + V_{loop}(h)]\Big|_{h=v+\delta v} = 0 \ , \tag{28}$$

Suppose higgs potential $V(h)$ is expanded as

$$V_{tree}(h) = V_{tree}(v) + \frac{1}{2}m^2(h-v)^2 + \frac{1}{6}am(h-v)^3 + \frac{1}{12}b(h-v)^4 + \dots \ , \tag{29}$$

and

$$V_{loop}(h) = V_{loop}(v) + \delta c \cdot m^3(h-v) + \frac{1}{2}\delta m^2(h-v)^2 + \frac{1}{6}\delta am(h-v)^3 + \frac{1}{12}\delta b(h-v)^4 + \cdots \tag{30}$$

thus, with (29), (30) and (28), we are left with following equation of the loop correction for vev

$$\frac{1}{3}(b+\delta b)y^3 + \frac{1}{2}(a+\delta a)y^2 + (1+\frac{\delta m^2}{m^2})y + \delta c = 0 \ , \tag{31}$$

where $y \equiv \delta v/m$, to simplify the solution of this cubic equation, we introduce some expansion parameter which will decompose the solution order by order; one reasonable assumption is to impose some small 'λ' dependence on these the couplings in eq.(31), which is the characteristic coupling strength in SM higgs potential.

$$a \sim \lambda^{1/2} \quad , b \sim \lambda^1, \quad m^2 \sim \lambda^0 \tag{32}$$

and on the one loop corrections

$$\delta a \sim \lambda^{3/2} \quad \delta b \sim \lambda^2, \quad \delta m^2 \sim \lambda^1 \quad \delta c \sim \lambda^{1/2} . \tag{33}$$

We comment that this kind of λ dependence is naturally supported by our power counting rule, i.e., the assumption of smaller coefficients of higher order operators in the low energy expansion because m^2, am and b are coefficients of h^2, h^3 and h^4, respectively. In next section (IV A) we will see these three terms belong to p^4, p^6 and p^8 orders in our higgs power counting system. With (32) and (33), the leading order of λ dependence of δv is found to be $\lambda^{1/2}$, which is again a natural result. Substitute them into (31), we find

$$\frac{1}{3}\delta b y^3\lambda^{7/2} + \frac{1}{3}by^3\lambda^{5/2} + \frac{1}{2}\delta a y^2\lambda^2 + (\frac{1}{2}ay^2 + \frac{\delta m^2}{m^2}y)\lambda^{3/2} + (y + \delta c)\lambda^{1/2} = 0 . \tag{34}$$

The leading order of λ of this equation is 1/2, thus,

$$y = -\delta c . \tag{35}$$

Detail one loop calculation gives

$$\delta c = \frac{a}{32\pi^2}(\frac{1}{\epsilon} - \gamma + 1 + \ln\frac{4\pi\mu^2}{m^2}) \qquad \delta v = -\frac{a\,m}{32\pi^2}(\frac{1}{\epsilon} - \gamma + 1 + \ln\frac{4\pi\mu^2}{m^2}) . \tag{36}$$

Beyond leading order of λ counting, one interesting case is $v|_{tree} = 0$, $a = 0$ and $m = 0$, which represent the situation that electroweak symmetry donot violates at tree level of EEWCL. E.q.(31) in this situation becomes

$$\frac{1}{3}(b + \delta b)y^3 + \frac{\delta m^2}{m^2}y = 0 . \tag{37}$$

Beside the trivial solution of $y = 0$, the nonzero leading dependence of λ is $y \sim \lambda^0$,

$$y(\frac{1}{3}by^2 + \frac{\delta m^2}{m^2})\lambda^1 + \frac{1}{3}\delta b y^3\lambda^2 = 0 , \tag{38}$$

which gives

$$(\delta v)^2 = -\frac{3\delta m^2}{b} . \tag{39}$$

This is actually the theory of massless quadratically self-interacting meson field by S.Coleman and E.Weinberg in Ref. [22]. It is quite straightforward using the following calculation result (53) to check (39) agrees with the result in Ref. [22]. Since $\delta m^2 \propto b$, (39) tells that this one loop vev radiative correction is independent of the self-interacting coupling as long as it is small enough to allow the perturbation. However, this massless mode is not included in following of our discussion because mass term in general case is more important than the self-interacting parts of higgs potential according to our higgs power counting.

A. higgs power counting

Before going any further, we turn back to the question raised in last section about the power counting problem of higgs field. It's condensation v is disconnected to any external source with explicit order, hence in order to include into theory all information from v, it should be counted as order of p^0 . The physical part of higgs interacts with external sources, hence it should be counted as at least p^2 order. That is to say,

$$h^{(0)} = v + \delta v, \qquad \tilde{h} = h^{(2)} + h^{(4)} + \dots . \tag{40}$$

This is consistent with the power counting in Ref. [9, 23], where the scalar source is also counted as p^2 order. On the other hand, because we care contributions from heavy higgs up to p^4 order, to which the only way of $h^{(4)}$ to contribute is in a linear form, which vanishes due to the vacuum condensation condition (the contribution to p^2 EWCL coupling from $h^{(2)}$ vanishes due to the same reason).

From now on we use h to denote $\tilde{h}$, define $x \equiv h/m$. The Lagrangian (23) is reparameterized as

$$\begin{aligned}
\mathcal{L}_{EEWCL} &= \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m^2(1-a\delta c)h^2 - m^4\tilde{V}(x) \\
&+m^2F_1(x)A_\mu^2 + m^2F_2(x)Tr(V_\mu^2) \\
&+G_0^i(x)l_4^i + G_1^j(x)(\partial_\mu x)l_3^{j\mu} + \frac{1}{2}G_2^k(x)(\partial_\mu x)(\partial_\nu x)l_2^{k\mu\nu} \\
&+G_3(x)(\partial_\mu x)^2(\partial_\nu x)A^\nu + G_4(x)(\partial_\mu x)^4 + \frac{1}{2}m^{-2}G_5^l(x)(\partial_\mu x)(\partial_\nu x)l_4^{l\mu\nu} .
\end{aligned} \tag{41}$$

Where

$$\begin{aligned}
\tilde{V}(x) &= \frac{1}{6}amx^3 + \frac{1}{12}bx^4 + \dots \\
F_1(x) &= (f_1 - f_3\delta c) + (f_3 - f_5\delta c)x + f_5x^2 + \dots \\
F_2(x) &= (f_2 - f_4\delta c) + (f_4 - f_6\delta c)x + f_6x^2 + \dots \\
G_\alpha(x) &= (g_\alpha - g'_\alpha\delta c) + (g'_\alpha - g''_\alpha\delta c)x + \frac{1}{2}(g''_\alpha - g'''_\alpha\delta c)x^2 + \dots, \qquad \alpha = 0,1,\dots,5
\end{aligned} \tag{42}$$

$f_i, i = 1,\dots,6$ and $g_\alpha, \alpha = 0,1,\dots,5$ are functions of v, all of them are free parameters because v is free, and terms containing δc denote loop correction from vev.

After the renormalization procedure is performed, all of the quantum corrections from higher order operators are included, we have such following equation of motion given by p^4

Lagrangian,

$$0 = \delta\mathcal{L}^{(4)} \Rightarrow \alpha h_c^{(2)} + \beta A_\mu^2 + \gamma Tr(V_\mu^2) = 0 \; , \tag{43}$$

where α, β, γ are renormalized parameters, with which we get the classic solution of higgs. Also, we have a similar equation from some higher p^n order,

$$0 = \delta\mathcal{L}^{(n+2)} \Rightarrow \partial_\mu\partial^\mu h^{(n-2)} \sim h^{(n)} + \text{other } \mathrm{p}^\mathrm{n} \text{ terms} \; . \tag{44}$$

On the other hand, note that p^2 Lagrangian is disconnected to higgs, it gives a p^2 equation of motion for V_μ[3].

$$D_\mu V^\mu = A\left[\partial_\mu Tr(TV^\mu)\right]T + BTr(TV_\mu)[V^\mu, T] + O(p^4) \; , \tag{45}$$

A, B are some coefficients, from this equation of motion, one obtain

$$[\partial_\mu Tr(TV^\mu)] = Tr(D_\mu(TV^\mu)) = Tr([V_\mu, T]V^\mu) + Tr(TD_\mu V^\mu) = 2A\left[\partial_\mu Tr(V^\mu)\right] \quad , \tag{46}$$

which leads to

$$\partial_\mu A^\mu = O(p^4) \; . \tag{47}$$

(44) and (47) are two equations of motion we use to simplify higher order operators.

The difference between this higgs power counting rule and Weinberg's power counting scheme lies in the fact that operators in higher order would contribute to lower orders through loop correction. This might indicate that higgs loop contribution to EWCL should be rather small compared with its tree level values. Note that 1-loop contribution from G_3 in the low energy expansion vanishes using (47). Up to p^4 order, One loop contribution from G_1 coupling is a total differential term. One loop contribution from G_5 could be totally represented by G_0. There's no 1-loop contribution from G_4. We arrange all independent terms order by orders as follows which could contribute to p^2 or p^4 order EWCL coefficients,

$$\begin{aligned}
\mathcal{L}^{(2)} &= m^2\left[(f_1 - f_3\delta c)A_\mu^2 + (f_2 - f_4\delta c)Tr(V_\mu^2)\right] \\
\mathcal{L}^{(4)} &= -\frac{1}{2}m^2(1 - a\delta c)h^2 + mh\left[(f_3 - f_5\delta c)A_\mu^2 + (f_4 - f_6\delta c)Tr(V_\mu^2)\right] + [g_0^i - (g_0^i)'\delta c]l_4^i \\
\mathcal{L}^{(6)} &= \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{6}amh^3 + \frac{1}{2}f_5 h^2 A_\mu^2 + \frac{1}{2}f_6 h^2 Tr(V_\mu^2) \\
\mathcal{L}^{(8)} &= -\frac{1}{12}bh^4 + \frac{1}{6}m^{-1}h^3[f_7 A_\mu^2 + f_8 Tr(V_\mu^2)] + \frac{1}{2}(g_0^i)'' h^2 l_4^i + \frac{1}{2}g_2^k m^{-2}(\partial_\mu h)(\partial_\nu h)l_2^{\mu\nu} \\
\mathcal{L}^{(10)} &= \frac{1}{2}(g_2^k)' m^{-3}(\partial_\mu h)(\partial_\nu h)h l_2^{k\mu\nu} \; .
\end{aligned} \tag{48}$$

We will see that beyond order of p^{10}, it is impossible that 1-loop higgs corrections make contributions to p^2 and p^4 order EWCL. When we are considering 1-loop corrections to p^2 and p^4 orders, all the coefficients of higher order operators take values at tree level. In (48), there are 19 free parameters undetermined, which are listed in Table I in more detail.

parameters		term	order	process
m	higgs mass	$-\frac{1}{2}m^2h^2$	4	higgs mass term
μ	present energy scale	$\ln\frac{\mu^2}{m^2}$		
f_1	coupling	$Tr(TV_\mu)Tr(TV^\mu)$	2	Z mass term
f_2	coupling	$Tr(V_\mu V^\mu)$	2	W^+W^- and Z mass term
f_3	coupling	$hTr(TV_\mu)Tr(TV^\mu)$	4	$h\to ZZ$
f_4	coupling	$hTr(V_\mu V^\mu)$	4	$h\to W^+W^-$ and ZZ
g_0^i	coupling	l_4^i	4	EWCL
$(g_0^i)'$	coupling	hl_4^i	6	EWCL
$\frac{1}{2}f_5$	coupling	$h^2Tr(TV_\mu)Tr(TV^\mu)$	6	$hh\to ZZ$
$\frac{1}{2}f_6$	coupling	$h^2Tr(V_\mu V^\mu)$	6	$hh\to W^+W^-$ and ZZ
$\frac{1}{6}a$	coupling	h^3	6	three higgs interaction
$\frac{1}{6}f_7$	coupling	$h^3Tr(TV_\mu)Tr(TV^\mu)$	8	$hhh\to ZZ$
$\frac{1}{6}f_8$	coupling	$h^3Tr(V_\mu V^\mu)$	8	$hhh\to W^+W^-$ and ZZ
$\frac{1}{12}b$	coupling	h^4	8	four higgs interaction
g_2^1	coupling	$\partial_\mu h\partial_\nu hTr(TV^\mu)Tr(TV^\nu)$	8	$hh\to ZZ$
g_2^2	coupling	$\partial_\mu h\partial_\nu hTr(V^\mu V^\nu)$	8	$hh\to W^+W^-$ and ZZ
$\frac{1}{2}(g_0^i)''$	coupling	$h^2l_4^i$	8	$hh\to EWCL$
$(g_2^1)'$	coupling	$h\partial_\mu h\partial_\nu hTr(TV^\mu)Tr(TV^\nu)$	10	$hhh\to ZZ$
$(g_2^2)'$	coupling	$h\partial_\mu h\partial_\nu hTr(V^\mu V^\nu)$	10	$hhh\to W^+W^-$ and ZZ

TABLE I: The summary of parameters appeared in EEWCL

B. integrate out higgs up to 1-loop

Different functional approaches of integrating out a scalar field are used in Ref. [24, 25, 26]. To integrate out higgs, we use the method of loop expansion in Ref. [12], but for our

purpose now, the procedure will be slightly different. In the traditional loop expansion to integrate out some field ϕ, one first calculate the effect action loop by loop, then use the solution of the stationery equation for this effective action to subtract out ϕ field in the theory. Infinities originated from loop integration will appear in the classic solution of the stationery equation, which has to be renormalized. Technically these infinities could be canceled by the corresponding counter terms in the theory after ϕ is integrated out. However, this kind of renormalization does not involve counter terms of the original theory and thus loses information of loop corrections from couplings denoting interactions between ϕ field and the rest part of the theory, i.e., all the loop corrections are represented by these couplings in the secondary theory after ϕ is integrated out. Even we achieve to absorb all divergences from loops of ϕ field into redefinitions of these secondary coefficients, the renormalized coefficients are only related to bare ones in original theory with ϕ. To explicitly conserve contributions from the original couplings of interactions involving ϕ field, both at tree level and at loop level, we consider into theory the renormalization effects of the couplings in original theory involving ϕ field which should play their own roles in cancellation of divergences. According to above analysis, we alter the procedure of integrating out higgs by interchanging computation of searching the solution of the stationery equation and of performing renormalization, i.e. first we do 1-higgs-loop renormalization of EEWCL (41), then get the classic solution of the stationery equation of the effective action, substitute it back into the Lagrangian, we complete the 1-loop integration of higgs. In present procedure, all divergences from loops can be absorbed into renormalized coefficients of EEWCL, which represent physical interactions among gauge bosons, goldstones and higgs.
In our computation system, we take low energy expansion which assumes smaller coefficients of higher order operators. On the other hand, these higher order operators contribute through loop correction, which would receive an extra loop factor of $1/16\pi^2$, hence makes the contribution from loops even smaller. This is the reason we only take one loop approximation in the following of calculations with higher loops effect omitted which will greatly simplifies our computation work.

Effective action up to 1-loop is given by

$$\Gamma^{1loop} = \int d^4x\mathcal{L}_{EEWCL} + \frac{i}{2}\ln Det\hat{D} \ . \tag{49}$$

What remains to be done to include the 1-loop contribution in the low energy expansion is to evaluate the determinant of the differential operator $\hat{D}$, which is defined as

$$\hat{D}(x,y) \equiv \frac{\delta^2 S}{\delta h(x)\delta h(y)} \ , \tag{50}$$

where $\hat{D}$ is given by,

$$D = -(\partial^2 + m^2 - A + C^{\mu\nu}\partial_\mu\partial_\nu) \ , \tag{51}$$

Where A and C are operators containing p^2 and p^4 contributions,

$$\begin{aligned} A &= -amh + f_5 A_\mu^2 + f_6 Tr(V_\mu^2) - bh^2 + f_7 m^{-1} h A_\mu^2 + f_8 m^{-1} h Tr(V_\mu^2) + m^{-2}(g_0^i)'' l_4^i \\ C^{\mu\nu} &= g_2^k m^{-2} l_2^{k\mu\nu} + (g_2^k)' m^{-3} h l_2^{k\mu\nu} \ , \end{aligned} \tag{52}$$

We take dimensional regularization scheme to do the one loop calculation, this choice can make us free of power type divergence terms which can avoid the possible fault estimations [27] and their disturbance to our power countings. Then

$$\begin{aligned} \frac{i}{2}\ln Det\hat{D} &= \int d^4x \int \frac{\mu^{4-D} d^D k}{(2\pi)^D} \langle x|k\rangle \ln det\hat{D}(\partial)\langle k|x\rangle \\ &= \frac{i}{2}\int d^4x \int \frac{\mu^{4-D} d^D k}{(2\pi)^D} [tr\ln\hat{D}(\partial + ik) - \ln(k^2 - m^2)] + Const \\ &= \frac{i}{2}\int d^4x \int \frac{\mu^{4-D} d^D k}{(2\pi)^D} \ln\left(1 + \frac{-\partial^2 - C^{\mu\nu}\partial_\mu\partial_\nu - 2ik_\mu(C^{\mu\nu} + g^{\mu\nu})\partial_\nu + C^{\mu\nu}k_\mu k_\nu + A}{k^2 - m^2}\right) \\ &= \frac{i}{2}\int d^4x \int \frac{\mu^{4-D} d^D k}{(2\pi)^D} \left[\frac{A + C^{\mu\nu}k_\mu k_\nu}{k^2 - m^2} - \frac{(A + C^{\mu\nu}k_\mu k_\nu)^2}{2(k^2 - m^2)^2} + O(p^6)\right] \\ &= \frac{1}{32\pi^2}\Big[-(L+1)m^2 A - \frac{(L+3/2)m^4}{4} C_\mu^\mu + \frac{L}{2}A^2 \\ &\quad + \frac{(L+1)m^2}{2} A C_\mu^\mu + \frac{(L+3/2)m^4}{16}\left[(C_\mu^\mu)^2 + 2(C^{\mu\nu})^2\right] + O(p^6)\Big] \ , \end{aligned} \tag{53}$$

In which L is defined as

$$L \ \equiv \ \frac{1}{\epsilon} - \gamma + \ln\frac{4\pi\mu^2}{m^2} \tag{54}$$

μ is energy scale appeared specially in dimensional regularization, A_2, A_4 and C_2, C_4 denote p^2 and p^4 order parts in A and C, respectively. In above calculation following conventions are used,

$$2\epsilon = 4 - D \qquad \Gamma(z) = \int dt e^{-t} t^{z-1} \qquad \Gamma(z+1) = z\Gamma(z) \qquad \Gamma(\epsilon) \ = \ \frac{1}{\epsilon} - \gamma + O(\epsilon) \ . \tag{55}$$

1-loop renormalized result for p^2 and p^4 order Lagrangian are listed below:

- p^2

$$\begin{aligned}\mathcal{L}^{(2)} &= m^2\left[\bar{f}_1 A_\mu^2 + \bar{f}_2 Tr(V_\mu^2)\right] \\ &= m^2\left[(f_1+\delta f_1)A_\mu^2 + (f_2+\delta f_2)Tr(V_\mu^2)\right] \ ,\end{aligned} \tag{56}$$

$$\begin{aligned}\bar{f}_1 = f_1 + \frac{1}{32\pi^2}\left[-\frac{(L+3/2)}{4}g_2^1 - (L+1)(f_5+af_3)\right] \\ \bar{f}_2 = f_2 + \frac{1}{32\pi^2}\left[-\frac{(L+3/2)}{4}g_2^2 - (L+1)(f_6+af_4)\right] \ ,\end{aligned} \tag{57}$$

- p^4

$$\begin{aligned}\mathcal{L}^{(4)} &= -\frac{1}{2}m_h^2 h^2 + m\bar{f}_3 l_4^A + \bar{f}_4 l_4^v + \bar{g}_0^i l_4^i \\ &= -\frac{1}{2}(m^2+\delta m^2)h^2 + m(f_3+\delta f_3)l_4^A + (f_4+\delta f_4)l_4^v + (g_0^i+\delta g_0^i)l_4^i \ ,\end{aligned} \tag{58}$$

In which l_4^i, $i = 1, 2, \ldots, 14$ denote the fourteen p^4 order EWCL operators.

$$\begin{aligned}m_h^2 &= m^2\left[1 - \frac{1}{16\pi^2}(L+1)(a^2+b) - \frac{a^2}{32\pi^2}\right] \\ \bar{f}_3 &= f_3 + \frac{1}{32\pi^2}\left[-(L+1)f_7 - \frac{L+3/2}{4}(g_2^1)' - \frac{L+1}{2}ag_2^1 - (2L+1)af_5\right] \\ \bar{f}_4 &= f_4 + \frac{1}{32\pi^2}\left[-(L+1)f_8 - \frac{L+3/2}{4}(g_2^2)' - \frac{L+1}{2}ag_2^2 - (2L+1)af_6\right]\end{aligned} \tag{59}$$

$$\begin{aligned}\bar{g}_0^4 &= g_0^4 + \frac{1}{32\pi^2}\left[-(L+1)\left[(g_0^4)'' + a(g_0^4)'\right] + \frac{L+3/2}{8}(g_2^2)^2\right] \\ \bar{g}_0^6 &= g_0^6 + \frac{1}{32\pi^2}\left[-(L+1)\left[(g_0^6)'' + a(g_0^6)'\right] + \frac{L+3/2}{4}g_2^1 g_2^2\right] \\ \bar{g}_0^5 &= g_0^5 + \frac{1}{32\pi^2}\left[-(L+1)\left[(g_0^5)'' + a(g_0^5)'\right] + \frac{L}{2}(f_6)^2 + \frac{L+1}{2}f_6 g_2^2 + \frac{L+3/2}{16}(g_2^2)^2\right] \\ \bar{g}_0^7 &= g_0^7 + \frac{1}{32\pi^2}\left[-(L+1)\left[(g_0^7)'' + a(g_0^7)'\right] + Lf_5 f_6 + \frac{L+1}{2}(f_5 g_2^2 + f_6 g_2^1) + \frac{L+3/2}{8}g_2^1 g_2^2\right] \\ \bar{g}_0^{10} &= g_0^{10} + \frac{1}{32\pi^2}\left[-(L+1)\left[(g_0^{10})'' + a(g_0^{10})'\right] + \frac{L}{2}(f_5)^2 + \frac{L+1}{2}f_5 g_2^1 + \frac{3(L+3/2)}{16}(g_2^1)^2\right]\end{aligned} \tag{60}$$

$$\begin{aligned}\bar{g}_0^i &= g_0^i + \frac{1}{32\pi^2}\left[-(L+1)\left[(g_0^4)'' + a(g_0^i)'\right]\right] \\ i &= 1, 2, 3, 8, 9, 11, 12, 13, 14\end{aligned} \tag{61}$$

All those $\delta f_i = \bar{f}_i - f_i$ and $\delta g_0 = \bar{g}_0 - g_0$ terms denote 1-loop level contributions to p^4 order EWCL couplings from higher order interactions among higgs, goldstones and gauge bosons, while p^4 operators in (58) can only contribute to p^4 order EWCL couplings at tree level. From (58), we get the stationery equation for h_c,

$$h_c = \frac{m}{m_h^2}[\bar{f}_3 Tr(TV_\mu)Tr(T^\mu) + \bar{f}_4 Tr(V_\mu V^\mu)] \ . \tag{62}$$

Note this solution is expressed by the 1-higgs-loop renormalized coefficients of the theory before higgs is integrated out. Substitute h_c back into (58), we get the tree level contribution from p^4 order higgs interacting with gauge bosons,

$$\mathcal{L}_{EWCL}^{(4)} \ = \ \frac{m^2}{2m_h^2}\left[(\bar{f}_3)^2 l_4^{10} + 2\bar{f}_3\bar{f}_4 l_4^7 + (\bar{f}_4)^2 l_4^5\right] \ . \tag{63}$$

Hence, after higgs is integrated out at 1-loop level in the low energy expansion up to p^4, we find out following final EWCL which include contributions from higgs interactions,

$$\mathcal{L}_{EWCL} \ = \ m^2\left[\bar{f}_1 Tr(TV_\mu)Tr(TV^\mu) + \bar{f}_2 Tr(V_\mu^2)\right] + (g_0^i + \Delta g_0^i) l_4^i \ , \tag{64}$$

with Δg_0^i denoting the contribution from integrating out higgs,

$$\begin{aligned}
\Delta g_0^5 \ &= \ \frac{1}{2}f_4^2 + f_4\delta f_4 - \frac{1}{2}(f_4)^2\frac{\delta m^2}{m^2} + \delta g_0^5 \\
\Delta g_0^7 \ &= \ f_3 f_4 + f_3\delta f_4 + f_4\delta f_3 - f_3 f_4\frac{\delta m^2}{m^2} + \delta g_0^7 \\
\Delta g_0^5 \ &= \ \frac{1}{2}f_3^2 + f_3\delta f_3 - \frac{1}{2}(f_3)^2\frac{\delta m^2}{m^2} + \delta g_0^{10} \\
\Delta g_0^j \ &= \ \delta g_0^j, j \neq 5, 7, 10 \ .
\end{aligned} \tag{65}$$

In following discussion we will use the redefined $\tilde{g}_0^i$ containing full contribution from integrating out higgs up to one loop level,

$$\tilde{g}_0^i \equiv g_0^i + \Delta g_0^i \tag{66}$$

In (65), terms with δ represent one loop contribution, which should be small, since they appear as a product of the loop factor $1/16\pi^2$ and higher order coefficients. Since we focus on the effects of higgs, the loop calculation does not include radiative corrections from other particles existed in the theory, the gauge bosons and goldstone bosones are totally viewed as a classic external source, i.e., there's no consideration about internal line or loops of gauge bosons and goldstone bosons. Loop effects from these particles can be viewed as backgrounds when we compare how higgs and some other new particle interactions will alter EWCL couplings through loops. Those contributions of other possible new particles will be investigated in separated papers.

V. DISCUSSIONS AND SUMMARY

Before starting the detail discussions, we first make some general analysis. First the coefficients in EWCL have their bare values g_0^i and corrections Δg_0^i from higgs. The bare part g_0^i is given in original EEWCL (48) and is independent of physics related to higgs. Since this paper focus on the higgs contributions part, we need to invent some arguments judging the smallness for values of those bare coefficients. We take assumption that higgs in reality will *really be* the next new particle we find in future experiment, then we could expect that it must plays an important role in the physics just below its threshold which implies at this scale some effects from bare coefficients are small compare to those from higgs. It is under this assumption, the contribution to some coefficients from higgs should be larger than its bare part: $\Delta g_0^i \gg g_0^i$ for some i and then we can ignore the corresponding bare part contributions.

From (59), (60), (61), (63) and (64), we see higgs contribution to EWCL coefficients starts from p^4 operators at tree level, and higher order operators contribute in two ways: 1.induce direct corrections through higgs loop, 2. induce correction to p^4 Lagrangian (58). If we omit all the loop corrections, we see higgs contribution concentrates in three terms: $\mathcal{L}_5$,$\mathcal{L}_7$ and $\mathcal{L}_{10}$, from $h \to ZZ$ and $h \to W^\pm$ decay. Thus, in tree level estimation, these two channels are important in phenomenology to find out the connection between higgs and EWCL couplings. A similar conclusion is presented in Ref. [28]. The most important loop correction comes from those p^6 operators with couplings f_5, f_6 and a, the first two couplings include $hh \to ZZ$ and $hh \to W^\pm$ scattering process, a is the coupling of three higgs self-interaction. From (58) and (59), we see f_5, f_6 and a all contribute through loop correction to p^4 operators $h[Tr(TV_\mu)]^2$ and $hTr(V_\mu V^\mu)$, which contribute to $\mathcal{L}_5$,$\mathcal{L}_7$ and $\mathcal{L}_{10}$ at tree level; from (60) we see f_5, f_6 contribute directly to $\mathcal{L}_5$,$\mathcal{L}_7$ and $\mathcal{L}_{10}$. Hence these three EWCL couplings should be most important in phenomenology when testing higgs signals connected to EWCL. Further, (57) tells us that p^2 EWCL operators in (56) are altered through loop correction by g_2^i, f_5+af_3 and f_6+af_4, involving $h, hh \to VV$ and three higgs self interaction process. Since f_6 gives correction to f^2 in $-\frac{f^2}{4}Tr(V_\mu V^\mu)$, f_5 gives correction to $\beta_1 f^2$ in $\frac{\beta_1 f^2}{4}[Tr(TV_\mu)]^2$, combine them together we can fix β_1 which is related to T parameter given by M.E.Peskin and T.Takeuchi [4] through relation $\alpha T = 2\beta_1$ [3], f_5 and f_6 might alter the value of T through higgs loop correction.

A. higgs decay and four-gauge-boson coupling

f_3 and f_4 are related to couplings g_{hWW} and g_{hZZ} in (48). Take unitary gauge, they are given by,

$$
\begin{aligned}
g_{hWW} &= -\frac{e^2}{s^2} f_4 \\
g_{hZZ} &= -\frac{e^2}{s^2 c^2}(f_3 + \frac{1}{2} f_4)
\end{aligned}
\tag{67}
$$

And the partial decay width of higgs is given by

$$
\begin{aligned}
\Gamma_{h\to ZZ} &= \frac{(2f_3+f_4)^2 e^4 m_h}{32 s^4 c^4}(1-\frac{4m_Z^2}{m_h^2})^{\frac{1}{2}} \\
\Gamma_{h\to WW} &= \frac{(f_4)^2 e^4 m_h}{32 s^4}(1-\frac{4m_W^2}{m_h^2})^{\frac{1}{2}} ,
\end{aligned}
\tag{68}
$$

or

$$
\begin{aligned}
(f_4)^2 &= \frac{32 s^4}{e^4 m_h}(1-\frac{4m_W^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to WW} \\
f_3 f_4 &= \frac{8 s^4}{e^4 m_h}\left[c^4(1-\frac{4m_Z^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to ZZ} - (1-\frac{4m_W^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to WW}\right] \\
(f_3)^2 &= \frac{2 s^4}{e^4 m_h}\left[(1-\frac{4m_W^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to WW} - 2c^4(1-\frac{4m_Z^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to ZZ}\right. \\
&\quad \left. + c^8(1-\frac{4m_W^2}{m_h^2})^{\frac{1}{2}}(1-\frac{4m_Z^2}{m_h^2})^{-1}\frac{(\Gamma_{h\to ZZ})^2}{\Gamma_{h\to WW}}\right] ,
\end{aligned}
\tag{69}
$$

where $s = \sin\theta_w$ and $c = \cos\theta_w$. Approximation $f_3 \ll f_4$ is used, due to the fact that f_3 induces explicit custodial symmetry breaking, which is small in reality[1]. Substitute this result into (63), we get tree level estimation for the contribution from higgs decaying into

VV to four-gauge-boson coupling, i.e., l_4^5, l_4^7 and l_4^{10}

$$\begin{aligned}
\mathcal{L}_{EWCL}\Big|_{5,7,10} &= \frac{32s^4}{e^4 m_h}\Bigg[(1-\frac{4m_W^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to WW} - 2c^4(1-\frac{4m_Z^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to ZZ} \\
&\quad + c^8(1-\frac{4m_W^2}{m_h^2})^{\frac{1}{2}}(1-\frac{4m_Z^2}{m_h^2})^{-1}\frac{(\Gamma_{h\to ZZ})^2}{\Gamma_{h\to WW}}\Bigg][Tr(TV_\mu)Tr(TV_\nu)]^2 \\
&\quad + \frac{16s^4}{e^4 m_h}\Bigg[c^4(1-\frac{4m_Z^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to ZZ} - (1-\frac{4m_W^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to WW}\Bigg]Tr(V_\mu V^\mu)[Tr(TV_\nu)]^2 \\
&\quad + \frac{4s^4}{e^4 m_h}(1-\frac{4m_W^2}{m_h^2})^{-\frac{1}{2}}\Gamma_{h\to WW}[Tr(V_\mu V^\mu)]^2 \ . \qquad (70)
\end{aligned}$$

Thus, the coefficient of $l_4^5 \equiv [Tr(V_\mu V^\mu)]^2$ is most sensitive to the partial decay width of $h \to WW$. In the energy scale below higgs mass, (70) can be used to estimate the value of the coupling g_{hVV} or the partial width of $h \to VV$, once four-gauge-boson coupling is obtained in the lab.

B. higgs mass dependence of 1-loop correction

Higgs mass dependence of each dimension zero coefficient in (64) is determined by L with

$$m^2\frac{dL}{dm^2} = -1 \qquad (71)$$

The result is given in Table II. We can see only $\bar{f}_1$ and $\bar{f}_2$ accept p^6 order corrections. One loop correction for other couplings starts from p^8.

Now we turn to the higgs mass dependence of certain terms related to T and S parameters in (64).

$$\frac{d\bar{f}_1}{dm^2} = \frac{1}{32\pi^2}\frac{1}{m^2}\left[\frac{1}{4}g_2^1 + f_5 + af_3\right] \qquad (72)$$

$$\frac{d\bar{f}_2}{dm^2} = \frac{1}{32\pi^2}\frac{1}{m^2}\left[\frac{1}{4}g_2^2 + f_6 + af_4\right] \ . \qquad (73)$$

Because af_3,g_2^1 and af_4,g_2^2 are respectively higher in order than f_5 and f_6, it's reasonable to suppose $g_2^1, af_3 \ll f_5$, $(g_2^2), af_4 \ll f_6$, thus they are omitted. Since $-\beta_1 = \bar{f}_1/\bar{f}_2$,

$$\alpha m^2\frac{dT}{dm^2} = 2\frac{d\beta_1}{dm^2} = \frac{2}{\bar{f}_2^2}(\bar{f}_2\frac{d\bar{f}_1}{dm^2} - \frac{\bar{f}_1 d\bar{f}_2}{dm^2}) = \frac{1}{16\pi^2}\frac{1}{\bar{f}_2}[f_5 + \beta_1 f_6] \ . \qquad (74)$$

For a given value of higgs mass, since $\bar{f}_2 \equiv -f^2/4 < 0$[3], when

$$f_5 + \beta_1 f_6 < 0 \ . \qquad (75)$$

C	$16\pi^2 \frac{dC}{d\ln m}\|_{p^6}$	$16\pi^2 \frac{dC}{d\ln m}\|_{p^8}$	$16\pi^2 \frac{dC}{d\ln m}\|_{p^{10}}$	$16\pi^2 \frac{dC}{d\ln m}\|_{p^{12}}$
$\bar{f}_1$	f_5	$\frac{g_2^1}{4}$	af_3	
$\bar{f}_2$	f_6	$\frac{g_2^2}{4}$	af_4	
$\bar{f}_3$		f_7	$\frac{(g_2^1)'}{4}$	$2af_5$
$\bar{f}_4$		f_8	$\frac{(g_2^2)'}{4}$	$2af_6$
$\tilde{g}_0^4$		$(g_0^4)''$		$a(g_0^4)'$
$\tilde{g}_0^6$		$(g_0^6)''$		$a(g_0^4)'$
$\tilde{g}_0^5$		$(g_0^5)''$		$f_4f_8 + a(g_0^5)' - \frac{(f_6)^2}{2}$
$\tilde{g}_0^7$		$(g_0^7)''$		$f_3f_7 + f_4f_8 + a(g_0^7)' + f_5f_6$
$\tilde{g}_0^{10}$		$(g_0^{10})''$		$f_3f_7 + a(g_0^{10})' - \frac{(f_5)^2}{2}$
$\tilde{g}_0^i$		$(g_0^i)''$		$a(g_0^i)'$
C	$16\pi^2 \frac{dC}{d\ln m}\|_{p^{14}}$	$16\pi^2 \frac{dC}{d\ln m}\|_{p^{16}}$	$16\pi^2 \frac{dC}{d\ln m}\|_{p^{18}}$	$16\pi^2 \frac{dC}{d\ln m}\|_{p^{20}}$
$\bar{f}_1$				
$\bar{f}_2$				
$\bar{f}_3$	$\frac{ag_2^1}{2}$			
$\bar{f}_4$	$\frac{ag_2^2}{2}$			
$\tilde{g}_0^4$		$-\frac{(g_2^2)^2}{8}$		
$\tilde{g}_0^6$		$-\frac{g_2^1 g_2^2}{4}$		
$\tilde{g}_0^5$	$\frac{f_6 g_2^2}{2} - \frac{f_4 (g_2^2)'}{4}$	$2af_4f_6 + \frac{b(f_4)^2}{2} - \frac{(g_2^2)^2}{16}$	$\frac{af_4 g_2^2}{2}$	$-\frac{(af_4)^2}{2}$
$\tilde{g}_0^7$	$\frac{(f_5 g_2^1 + f_6 g_2^2)}{2} - \frac{[f_3 (g_2^2)' + f_4 (g_2^1)']}{4}$	$2a(f_3f_6 + f_4f_5) + bf_3f_4 - \frac{g_2^1 g_2^2}{8}$	$\frac{a(f_3 g_2^2 + f_4 g_2^1)}{2}$	$-af_3f_4$
$\tilde{g}_0^{10}$	$\frac{f_5 g_2^1}{2} - \frac{f_3 (g_2^1)'}{4}$	$2af_3f_5 + \frac{b(f_3)^2}{2} - \frac{3(g_2^1)^2}{16}$	$\frac{af_3 g_2^1}{2}$	$-\frac{(af_3)^2}{2}$
$\tilde{g}_0^i$				

TABLE II: The summary of higgs mass dependence of coefficients appeared in (64)

T parameter increases when higgs mass increases, or present energy scale decreases far away from higgs mass, which means a heavier higgs allows larger value of T parameter, which is consistent with SM data in Ref. [28, 29]. S parameter is related to g_0^1, which has mass dependence as

$$m^2 \frac{dS}{dm^2} = m^2 \frac{dg_0^1}{dm^2} = \frac{1}{32\pi^2}[(g_0^1)'' + a(g_0^1)'] \,. \tag{76}$$

SM data in Ref. [28, 29] tells smaller value of S is consistent with the existence of heavier higgs for better data fitting result. Thus $(g_0^1)'' + a(g_0^1)' < 0$.

C. higgs mass limit

For a given value of μ, here we test a assumption that when higgs mass goes to infinity, or when $\mu \ll m$, all the contributions from higgs loop to EWCL couplings should decrease when higgs mass increases. This is a reasonable assumption because the inference from higgs should become smaller and smaller when it is farer and farer away from present energy scale. That is to say the coefficients should have such mass dependence

$$\frac{d\delta C}{dm^2}\delta C < 0 \ . \tag{77}$$

However, this condition can only be satisfied when there is a factor of minus power of mass. From above one loop result we see there is no such term which accepts this restriction. This is due to the fact that the only dimensional constant in EEWCL is higgs mass m (the vacuum condensation v should be proportional to m), since classic solution of higgs field is expanded as

$$\tilde{h} \sim m^{-1}\mathcal{L}^{(2)} + m^{-3}\mathcal{L}^{(4)} + \ldots \tag{78}$$

Suppose a general form of EEWCL is written as

$$\mathcal{L}_{EEWCL} = \sum_n C_n m_h^x \tilde{h}^y \mathcal{L}^{2z} \tag{79}$$

with relation $x + y + 2z = 4$. $\mathcal{L}$ denotes the external source with momentum order p^{2z}. We know (79) can be expanded in momentum order as

$$\sum_n C_n (p^{2y+2z} + p^{4y+2z} + \ldots) \tag{80}$$

On the other hand, if we view it as a expansion of different operators in m^{-1},

$$\sum_n C_n \left[(\frac{1}{m})^{2y+2z-4} + (\frac{1}{m})^{4y+2z-4} + \ldots \right] \tag{81}$$

Up to p^4, we have the relation

$$O(m^{-1}) = O(p) - 4 \leq 0 \tag{82}$$

This means up to p^4 order there include only non-decoupling effect of heavy higgs, while decoupling effect in the higgs mass limit $m_h \to \infty$ would not show up until in p^6 and higher orders.

D. standard model higgs

Although the title of this paper is called nonstandard higgs in EWCL, since we have written down the most general form of EEWCL, conventional standard model higgs is certainly included in our theory. If we set as TABLE III and all other couplings become zero, we go

C	$\bar{f}_2$	$\bar{f}_4$	f_6	a	b	others
value	$-\frac{1}{32\lambda}$	$-\frac{\sqrt{2}}{8}\frac{1}{\sqrt{\lambda}}$	$-\frac{1}{2}$	$6\sqrt{2\lambda}$	12λ	0

TABLE III: The summary of coefficients appeared in SM.

back to SM higgs in (21). Then we can use result in previous subsections to get the result of integrating out SM higgs, which is listed in TABLE IV. Obviously higgs in SM cannot

C	$\bar{f}_2$	$\bar{f}_4$	$\tilde{g}_0^5$	others
$8\pi^2\frac{dC}{d\ln m}$	-1	$\frac{3}{2}$	$\frac{7}{32}$	0

TABLE IV: The summary of higgs mass dependence of coefficients appeared in SM.

be decoupled from the rest of theory even though it is very heavy. We see T parameter decreases when higgs mass increases in SM according to condition (75). And from Table III we see again the corrections have no dependence on λ.

In Summary, we have investigated the possible effects from single higgs to all p^2 and p^4 order coefficients in bosonic part of EWCL. We have included in all possible higgs couplings which may contribute to p^2 and p^4 order EWCL coefficients within one higgs-loop precision. We find three terms, $\mathcal{L}_5$, $\mathcal{L}_7$, $\mathcal{L}_{10}$ in EWCL are important, for which the contributions from higgs can be further expressed in terms of higgs partial decay width $\Gamma_{h\to ZZ}$ and $\Gamma_{h\to WW}$. Higg mass dependence of the coefficients are discussed and SM is one of special case in our discussion.

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